Collusion and Discrimination in Organizations: Comment

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Abstract

Ishiguro (2004) shows that discriminatory wage schemes are optimal among collusion-proof contracts under relative performance evaluation when agents collude. However, that analysis depends on the assumption that the agents cannot observe their performances. We investigate how optimal contracts should be modified when the agents observe the realized firm value. We show that the optimal collusion-proof contract can be a low-powered incentive scheme with buy-out options under some circumstances. Also, this suggests the frequent use of options in wage scheme as an optimal response to collusion.

Keywords: Collusion; Relative performance evaluation; Buy-out option; Moral Hazard

JEL classification: D20; J30; J70
1 Introduction

Relative performance evaluation can sometimes resolve the moral hazard problem by making the compensation of agents contingent on the relative ranking of their performance. However, such a scheme is vulnerable to collusion among agents as the relative ranking of their performance is not changed when they all shirk compared to when they all work. Ishiguro (2004) demonstrates that a discriminatory wage scheme will be introduced when only relative performance is available and collusion among agents is possible. To deter collusion, one agent is offered a higher-powered incentive scheme than the other agent even if they are equally productive ex ante. Such a discriminatory wage scheme creates a discrepancy between the interests of agents and helps to deter their collusion.

However, Ishiguro (2004) ignores the possibility of using the realized output value implicitly in the contract in order to induce high effort. Even if information about individual output and total output is not contractible, the principal can use this information structure to make herself better off. A collusion-proof contract using this information may be a mixture of low-powered wage scheme and a buy-out option with a fixed price. Unlike in Ishiguro (2004), all agents are symmetrically treated by the principal under this contract. However, it will be revealed that the optimality of a certain contract depends on the cost level of agents. For instance, a discriminatory scheme turns out to be optimal if the cost is not so small. Thus, the exact form of contract against collusion differs according to the cost level as well as the information structure.

A few studies, including Ishiguro (2004), deal with the properties of collusion-proof contracts in moral hazard environments when the absolute performance evaluation is not possible. Baliga and Sjöström (1998) show that delegation may be optimal when agents collude, if the agents work in sequence. In their model, agent 1’s effort level is known to both agents while agent 2’s effort is his private information. Thus, the agents are asymmetric ex ante, and the side-contracts can be directly contingent on agent 1’s effort. However, this paper allows only monetary side-transfers as effort levels of all agents are private information.

The current paper is similar to Che and Yoo (2001), which also establishes the optimality of low-powered group incentives in a different context. They show that the optimal incentive scheme displays many observed features of team-oriented organizations, in which there is long-term interaction among agents, while our model is short-run and static.

The next section lays out the basic features of the model. Section 3 studies contracts designed to deter collusion and analyzes their effectiveness with respect to agents’ cost level. Section 4 investigates the use of buy-out option in contract and shows that a discriminatory wage may not be an optimal way
of deterring collusion. Section 5 concludes.

2 The Model

Consider an organization that consists of three risk neutral players. A principal hires two agents A and B whose efforts stochastically yield some outputs to her. When agent \(i\) chooses effort level of \(e_i\), the output generated is

\[ y_i = e_i + \epsilon_i, \]

where \(\epsilon_i\) is a random shock. For simplicity, we restrict our attention to a two-effort case, so each agent chooses either high or low effort: \(e_i \in \{h, l\}\) for \(i = A, B\). The utility function of each agent is \(\omega - C(e)\) where \(\omega\) is monetary payoff and \(C(e)\) denotes the effort cost. It is assumed that \(C(h) = c > 0\) and \(C(l) = 0\). Also we assume that \(\epsilon_A\) and \(\epsilon_B\) are independent and identically distributed with distribution \(F\), where \(F(0) = 1/2\). The corresponding density function is denoted by \(f(\cdot)\) and satisfies \(f(x) > 0\) for \(x \in \mathbb{R}\). Finally, the reservation payoffs of all parties are assumed to be zero.

The effort level of each agent is purely private information. Moreover, it is assumed that the absolute measure of agents’ performances, \(y_A\) and \(y_B\) are not available or are too costly to use. Rather their relative ranking is verifiable and hence contractible.

The distribution function of \(y_i\) is given by \(P[y_i \leq y] = F(y - \epsilon_i)\) for \(i = A, B\). Also, the distribution function of \(\epsilon_i - \epsilon_j\) is

\[ G(\delta) \equiv P[\epsilon_i - \epsilon_j \leq \delta] = \int_{-\infty}^{\infty} F(\delta + \epsilon)f(\epsilon)d\epsilon. \]

Note that \(G(-\delta) = 1 - G(\delta)\).

Here, we follow the timing of the game in Ishiguro(2004) except that the realization of \(y_A + y_B\) is observable to the agents at Stage 5, although it is not contractible. Note that Stage 3 should be neglected in the collusion-free situation.

1. The principal offers an initial contract \(\omega\).
2. Each agent decides whether to accept it or not.
3. The agents may write a side-contract.
4. The agents choose their efforts simultaneously.
5. The outputs \(y_A\) and \(y_B\) are realized and final payments are made according to initial and side-contracts. The agents can negotiate over the exercise of a buy-out option after the outputs are realized. At this stage agents’ bargaining powers are exogenously given by \(\alpha\) and \(1 - \alpha\).
3 Contracts against Collusion

When players are risk-neutral, one way to make agents choose high effort levels is using a fixed rent scheme (or selling the firm contract). Under this scheme, \( \omega \) does not depend on \( y_A, y_B \), or relative rankings. Regardless of the outputs, the agents are obliged to pay a fixed amount of money to the principal. Thus, the agents are residual claimants who take all the risk associated with the firm’s value. The following table shows the expected payoff to the agents without considering the fixed rent, when they share the realized firm value equally.

Table 1: Expected Payoff to the Agents with Fixed Rent Scheme.

<table>
<thead>
<tr>
<th></th>
<th>( h )</th>
<th>( B )</th>
<th>( l )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td>( (h-c, h-c) )</td>
<td>( (h+l/2 - c, h+l) )</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td>( (h+l, h+l - c) )</td>
<td>( (l, l) )</td>
<td></td>
</tr>
</tbody>
</table>

Under the fixed rent scheme, the agents will choose \( H \) if \( c < \frac{h-l}{2} \). Thus, the agents are likely to choose high effort when the cost level is low relative to the productivity of effort, \( h-l \). If \( c > \frac{h-l}{2} \), \( L \) will be chosen by the agents. Note that inducing high effort is optimal when \( h-l > c \). Under the fixed rent scheme, one agent’s effort increases the expected value of the firm, but the agent only has a partial claim on the increased value. If \( \frac{h-c}{2} < c < h-l \), \( H \) cannot be induced even though high effort is optimal. So such a scheme does not solve the moral hazard problem fully. Note that the maximum of the fixed rent for the principal is \( 2h \) if \( c \leq \frac{h-l}{2} \), or \( 2l \) otherwise.\(^2\)

\(^2\)Here, we assumed that both of the agents will get the same share of the firm’s value. Suppose agents’ shares are \( \alpha \) and \( 1 - \alpha \) respectively, and let \( \alpha \leq \frac{1}{2} \) without loss of generality. Then, both of the agents will choose high effort if \( c < \alpha(h-l) \) and will choose low effort if \( c > (1-\alpha)(h-l) \). When \( \alpha(h-l) < c < (1-\alpha)(h-l) \), the agent with \( \alpha \) share will make low effort while the other agent will make high effort. So uneven share among agents make it more difficult to induce them to choose high effort. More detail will be discussed later in this paper.
Furthermore, this fixed rent scheme becomes infeasible once we introduce limited liability constraint. No matter what the fixed rent is, it is possible for the realized firm value to be even less.

**Figure 1: Cost level and Fixed Rent Scheme.**

(h, h) is optimal.  
(h, h) is feasible.  
(h, h) is infeasible.

With the limited liability constraints, Ishiguro (2004) shows that the optimal collusion-proof contract is a non-anonymous wage scheme, where one agent is offered a higher-powered incentive scheme than the other even when they are identical ex ante. In his model, it is publicly observable whether \( y_i > y_j \), for \( i, j = A, B, i \neq j \). Thus the contract the principal offers to agent \( i \) specifies the payments to him contingent only on whether \( y_i > y_j \). Let \( W_i \) and \( L_i \) denote the wages to be paid to agent \( i \) when \( y_i > y_j \) and when \( y_i < y_j \) respectively. The proposed optimal contract is \( W_i = c/\Delta P \) and \( L_i = W_j = L_j = 0 \), \( i, j = A, B, i \neq j \), where \( \Delta P \equiv G(0) - G(l - h) = \frac{1}{\lambda} = G(l - h) \), implements the asymmetric effort pair \((h, l)\). Note that this contract is optimal only if

\[
c < \tilde{c} \equiv (h - l) \cdot \frac{G(h - l) - 1/2}{G(h - l)} \leq \frac{h - l}{2}.
\]

Otherwise, the principal can obtain the expected payoff \( 2l \), implementing \( e = (l, l) \), which is greater than the expected payoff from that discriminatory contract, \( h + l - G(h - l) \cdot \frac{\Delta P}{\lambda} \).

**Figure 2: Discriminatory Scheme in Ishiguro (2004).**

\( (h, l) \) can be simply induced by low-powered wage scheme, \( W_i = L_i = 0 \) for \( i = A, B \).
Thus, the inefficiency due to limited liability constraints is two-fold. First, the limited liability constraints deter the implementation of effort levels \((h, h)\). Second, the constraint shrinks the range where the contract is valid: the relevant threshold cost level decreases from \((h - l)/2\) to \(\tilde{c}\).

4 Collusion-proof Contract with Buy-out Option

Consider the following scheme. The initial contract \(\omega\) specifies \(W_i\) and \(L_i\) for \(i = A, B\). Also, \(\omega\) endows the winner with a right to buy out the realized output at given price \(S\). But this option cannot be exercised unless the loser grants his permission. With this contract, the winner would like to exercise the option whenever the firm’s output value is greater than \(S\).

Thus, the principal’s expected payoff with this contract is

\[
\begin{align*}
\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S - e_A - e_B - y (e_A + e_B + x + y) f(x) df(y) dy \\
+ \int_{-\infty}^{\infty} \int_{S-e_A-e_B-y}^{\infty} S f(x) dx f(y) dy \\
- (W_A + L_B) \cdot P\{y_A > y_B\} - (W_B + L_A) \cdot P\{y_B > y_A\}.
\end{align*}
\]

Now, we can resort to the Collusion-Proof Principle\(^4\): the principal can restrict her attention to collusion-proof contract.\(^5\) For any contract which is not collusion proof, she can replicate the same outcome by some collusion-proof contract. We focus on the symmetric wage scheme, in which \(W_A = W_B\) and \(L_A = L_B\). This symmetry may lessen the probability of collusion. If \(W_i - W_j > L_i - L_j\), then agent \(i\)’s benefit from shirking by agent \(j\) is larger than agent \(j\)’s benefit from shirking by agent \(i\). In this event, agent \(i\) has a strict incentive to make a side transfer to agent \(j\), inducing him to shirk. By the same conjecture, the low-powered scheme would lessen the chance of collusion among agents. There’s no gain from shirking by peer when the wage scheme is low-powered for all agents. Thus, the following Lemma is immediate.

**Lemma 1** The symmetric, low-powered wage scheme minimizes the possibility of collusion among agents.


\(^5\)In present paper, I focus on the sets of buy-out contracts based on the relative performance, which may not be globally optimal among all possible contracts that uses the output levels of two agents without any restriction. I thank a referee for pointing out this.
Now, if $W_i = L_i = 0$ for $i = A, B$, then the payoff to the principal is

$$2h - \int_{-\infty}^{\infty} \int_{S-2h-y}^{\infty} (2h + x + y - S) dF(x) dF(y). \quad (1)$$

Recall that $\alpha$ and $1 - \alpha$ are the agents’ bargaining powers after exercising the option. Without loss of generality, we can suppose that $\alpha \geq 1/2$. Then, $(h, h)$ will be chosen if

$$\alpha \int_{-\infty}^{\infty} \int_{S-2h-y}^{\infty} (2h + x + y - S) dF(x) dF(y) - c$$

$$> \alpha \int_{-\infty}^{\infty} \int_{S-h-l-y}^{\infty} (h + l + x + y - S) dF(x) dF(y),$$

and

$$(1 - \alpha) \int_{-\infty}^{\infty} \int_{S-2h-y}^{\infty} (2h + x + y - S) dF(x) dF(y) - c$$

$$> (1 - \alpha) \int_{-\infty}^{\infty} \int_{S-h-l-y}^{\infty} (h + l + x + y - S) dF(x) dF(y).$$

Suppose the agents will choose $(h, h)$ under this contract. As $h + l - G(h - l) \cdot \frac{\Delta P}{\Delta T}$ is the expected payoff from the contract in Ishiguro(2004), the principal can obtain the higher payoff with above contract when

$$c < \frac{G(h - l) - 1/2}{G(h - l)} (h - l - L),$$

where $L \equiv \int_{-\infty}^{\infty} \int_{S-2h-y}^{\infty} (2h + x + y - S) dF(x) dF(y)$.

Therefore, $(h, h)$ will be implemented with above contract when $c$ is small; in particular, when

$$c < \hat{c} \equiv \min \left\{ \frac{G(h - l) - 1/2}{G(h - l)} \cdot (h - l - L), \min \{\alpha, 1 - \alpha\} \cdot (M - N) \right\},$$

where

$$M \equiv \int_{-\infty}^{\infty} \int_{S-2h-y}^{\infty} (2h + x + y - S) dF(x) dF(y),$$

and

$$N \equiv \int_{-\infty}^{\infty} \int_{S-h-l-y}^{\infty} (h + l + x + y - S) dF(x) dF(y).$$

*The wages cannot be negative due to the limited liability constraints.
Figure 3: Low-powered Wage Scheme with Option.

Thus, if the effort cost is sufficiently low (i.e., if $c < \hat{c}$), the principal will use the low-powered scheme with buy-out option, and $(h, h)$ is induced. The cut-off value $\hat{c}$ depends on $S$. Since the principal can control the value of $S$, and so of $\hat{c}$, she will do so in a way that maximizes her payoff (1).

The above argument yields our main result.

**Proposition 1** Suppose $c < \hat{c}$. Then the optimal collusion-proof contract is given by the low-powered wage scheme with buy-out option.

This contract can be easily shown to be collusion-proof. When the cost is intermediate ($\hat{c} < c < \bar{c}$), the optimal contract by the principal becomes discriminatory as in Ishiguro(2004) and $(h, l)$ will be induced. When $\bar{c} < c < h - l$, $(l, l)$ is induced by the principal, although high effort is socially optimal. This inefficiency is due to the limited liability constraints and the moral hazard problem in the model.

Under the scheme above, the cut-off value, $\hat{c}$, must be small when one agent has strong bargaining power (i.e., $\alpha$ or $1 - \alpha$ is close to zero). In other words, our scheme does not work when agents’ bargaining powers are asymmetrically distributed. In this case, we can introduce the following scheme:

After the total output is realized, one agent is selected by tossing a coin. That agent is given the option to buy the total output at price $S$.

5 Concluding Remarks

The possibility of collusion among agents can reduce the social welfare, especially when the output or performance is not contractible. Ishiguro(2004) shows that if only the relative ranking of the agents’ performances can be used in the contracts, only suboptimal outcomes can be implemented through discriminatory wage schemes. However, this paper demonstrates that the socially optimal effort level of agents can be induced by using the realized firm’s output implicitly in the contract. The payoff structure of the agents is similar to that of a bonus scheme. The key difference is that bonuses are based on the

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7I thank Shingo Ishiguro for suggesting this scheme.
verifiable measurement such as absolute performances, which is not available here.

References


