Comparative Statics for Oligopoly, Oligopsony and Oligopsonistic Oligopoly

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Abstract

Comparative static analysis is presented for three Cournot models, that is, Cournot oligopoly, Cournot oligopsony and Cournot oligopoly coexistent with Cournot oligopsony. Comparative statics for the first two models are made possible by taking into account the diagrams which determine simultaneously the industry quantity and product or factor price, and that for the third model by considering simultaneous determination of the equilibrium industry output and factor demand.

Keywords: Cournot; Comparative statics; Oligopoly; Oligopsony; Oligopsonistic oligopoly

JEL classification: D4, L1

1 Introduction

Mathematical analysis of the existence, uniqueness and stability of the Cournot oligopoly with or without product differentiation abounds. McManus (1962, 1964) has conducted a diagrammatic analysis of Cournot oligopoly. He has analyzed the existence and uniqueness as well as the effect of entry for Cournot oligopoly. Dixit (1986) has first

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given a systematic, comparative static analysis of Cournot oligopoly on the basis of the stability condition for the equilibrium. To the best of our knowledge, however, few economists have analyzed oligopsony and oligopoly coexistent with oligopsony (oligopsonistic oligopoly).

In this paper we will be concerned with the existence of the equilibrium and comparative statics for three models of imperfect competition, i.e. Cournot oligopoly where the factor market is perfectly competitive and the product market is oligopolistic, Cournot oligopsony where the product market is perfectly competitive and the factor market is imperfectly competitive in the sense that the factor supply depends on the factor price, and Cournot oligopsonistic oligopoly where the product market is oligopolistic and the factor market is oligopsonistic. Adapting the approach of Szidarovzky and Yakowitz (1977), and Okuguchi(1993), we will reduce the existence problems to fixed-point problems regarding the industry variables, that is, the industry output in Cournot oligopoly, the total industry demand for or supply of the factor of production in Cournot oligopsony, and the industry output and the total factor demand or supply in Cournot oligopsonistic oligopoly. As a consequence of this, we will be able to show the equilibrium price and quantity - the equilibrium industry output and product price in Cournot oligopoly; the equilibrium industry factor demand or supply and factor price in Cournot oligopsony ; the industry output and industry factor demand in Cournot oligopsonistic oligopoly - simultaneously in diagrams. We will find that our approach will enable us to derive our comparative static results easily without performing complicated computations as well as without explicitly taking into account *ad hoc* stability conditions for the Cournot equilibria.

2 Cournot Oligopoly

In this section n firms producing homogeneous goods and facing competitively given factor price are assumed to behave under the Cournot assumption as regards to rivals' outputs. Let firm $i's$ profit function π_i be defined as

$$\pi_i \equiv x_i p\left(\Sigma x_i\right) - C_i\left(x_i, \alpha_i\right), \quad i = 1, 2, \cdots, n, \tag{1}$$

where x_i and C_i are its output and cost function, α_i is a parameter affecting C_i, and the market price of the homogeneous goods p is a function of its total demand with $p^{'} < 0$. Throughout this paper we assume differentiability of the relevant functions up to the orders as necessary. We assume:

$$\frac{\partial C_i}{\partial \alpha_i} \equiv C_{i\alpha_i} \leq 0, \qquad \frac{\partial^2 C_i}{\partial x_i \partial \alpha_i} \equiv C^{'}_{i\alpha_i} \leq 0, \quad i = 1, 2, \cdots, n \tag{2}$$

with strict inequality for at least one i.

$$p^{'} \leq C_i^{''} \equiv \frac{\partial^2 C_i}{\partial x_i^2}, \qquad i = 1, 2, \cdots, n. \tag{3}$$

$$p^{'} + x_i p^{''} < 0, \qquad i = 1, 2, \cdots, n. \tag{4}$$

Let α_i denote firm $i^{'}s$ technologylevel with a higher α_i corresponding to a higher technology level. Then an increases in α_i leads to lower total and marginal costs of firm i, satisfying strict inequality in (2). If its marginal cost is constant or increasing, (3) holds. Under (4), we have $\frac{\partial^2 \pi_i}{\partial x_i \partial x_j} < 0, i \neq j$ Hence, any two firms' products are strategic substitutes for each other in the sense of Bulow *et al.* (1985). If the maximum is interior and if, in addition α_i is given, the first order condition for maximization of π_i with respect to x_i is given by

$$p\left(X\right) + x_i p^{'}\left(X\right) - C_i^{'}\left(x_i \alpha_i\right) = 0, \quad i = 1, 2, \cdots, n, \tag{5}$$

where $X \equiv \sum x_j$ is the industry output. We note that the second order condition is ensured by (3) and (4). Solving (5) with respect to x_i

$$x_i \equiv \varphi^i\left(X, \alpha_i\right), \quad i = 1, 2, \cdots, n, \tag{6}$$

where we have in view of (2), (3) and (4),

$$\frac{\partial \varphi^i}{\partial X} \equiv \varphi^i_x = -\frac{p^{'} + x_i p^{''}}{p - C_i^{''}} < 0, \quad i = 1, 2, \cdots, n. \tag{7.1}$$

$$\frac{\partial \varphi^i}{\partial \alpha_i} \equiv \varphi^i_\alpha = \frac{C^{'}_{i\alpha_i}}{p' - C_i^{''}} \geq 0, \quad i = 1, 2, \cdots, n.$$

with strict inequality for at least one i. (7.2)

Given $\alpha_i^{'}s$, the Cournot equilibrium industry output is identified with the unique solution (or fixed-point) of

$$X = \varphi\left(X, \alpha_1, \cdots, \alpha_n\right) \equiv \sum_j \varphi^j\left(X, \alpha_j\right), \tag{8}$$

where

$$\frac{\partial \varphi}{\partial X} = \sum_j \frac{\partial \varphi^j}{\partial X} < 0, \tag{9.1}$$

$$\frac{\partial \varphi}{\partial \alpha_i} = \sum_j \frac{\partial \varphi^j}{\partial \alpha_j} \geq 0, \quad i = 1, 2, \cdots, n$$

$$\text{with strict inequality for at least one } i. \tag{9.2}$$

The Cournot equilibrium industry output and product price are the unique solution of

$$p = p\left(X\right), \; p^{'} < 0 \tag{10.1}$$

and

$$p = p\left(\varphi\left(X, \alpha_1, \cdots, \alpha_n\right)\right) \equiv g\left(X, \alpha_1, \cdots, \alpha_n\right), \tag{10.2}$$

where

$$\frac{\partial g}{\partial X} = p^{'} \frac{\partial \varphi}{\partial X} > 0. \tag{11.1}$$

$$\frac{\partial g}{\partial \alpha_i} = p^{'} \frac{\partial \varphi}{\partial \alpha_i} \leq 0. \text{ with strict inequality for at least one } i. \tag{11.2}$$

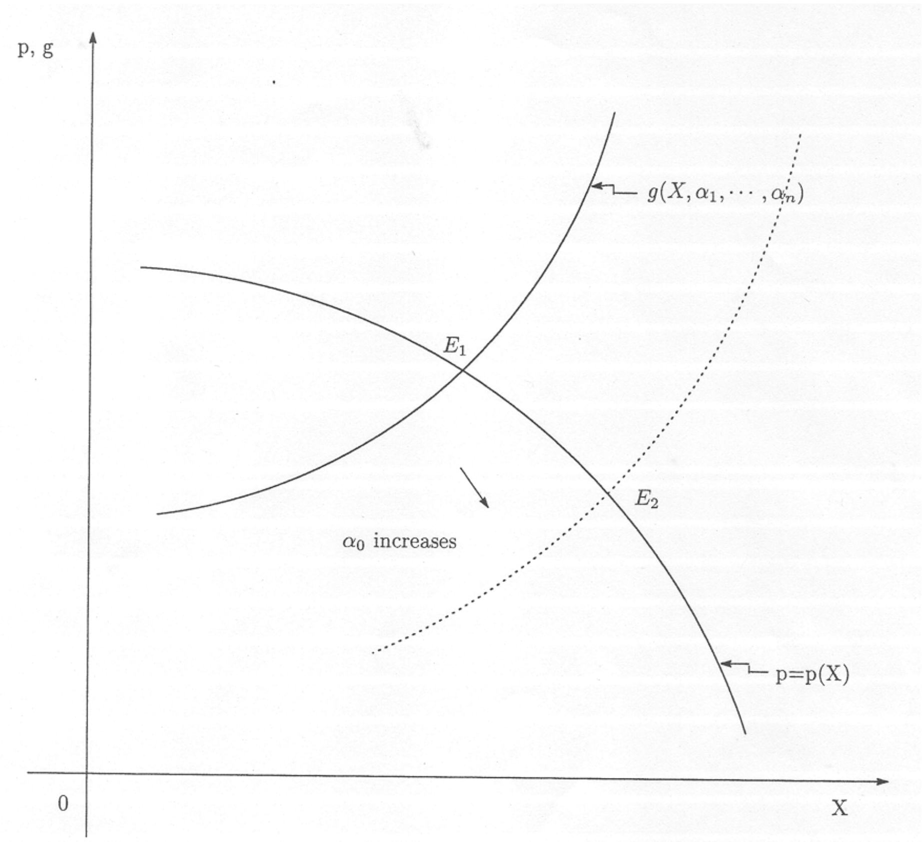
Given $\alpha_i^{'}s$, the unique solution is given by the intersection E_1 of the downward-sloping curve for (10.1) (the inverse demand function) and the upward-sloping curve for (10.2) as in Figure 1.

Now let the strict inequality in (2) hold for i_0. Then the industry supply curve shifts downward as shown by the dotted curve in Figure 1 but the demand curve remains intact, leading to the new intersection E_2, which gives rise to a larger equilibrium industry output and a lower product price.

Suppose next entry of a new firm, say, firm $n+1$, and assume (2), (3) and (4) to hold also for the entrant. The curve for

$$p\left(\varphi\left(X, \alpha_1, \cdots, \alpha_n\right) + \varphi^{n+1}\left(X, \alpha_{n+1}\right)\right)$$

Figure 1: Equilibrium Industry Output and Product Price.



lies below that for $p(\varphi(X, \alpha_1, \cdots, \alpha_n))$. Hence, the new industry supply curve shifts downward, resulting in a larger equilibrium industry output and a lower product price. This is the quasi-competitiveness of Cournot oligopoly as observed by Ruffin (1971), Okuguchi (1973, 1976, 1993) and Corchon (1994).

3 Cournot Oligopsony

In this section we consider oligopsony facing perfect competition in the product market. Let there be n oligopsonists producing homogeneous goods with the help of one factor of production, say, labor and suppose they sell their products in the same market. Firm $i's$ profit function is by definition

$$\pi_i \equiv pf_i(l_i, \alpha_i) - l_i w(L), \quad i = 1, 2, \cdots, n, \tag{12}$$

where p and w are the competitive product price and the wage rate as a function of the total labor supply L ; f_i is firm $i's$ production function, where l_i is firm $i's$ demand for labor; α_i is a parameter affecting the production function. The parameter may refer to the firm's technology level or the amount of fixed capital. We assume that the production and wage functions satisfy (13) and (14), respectively.

$$f_i' \equiv \frac{\partial f_i}{\partial l_i} > 0, \; f_i'' \equiv \frac{\partial^2 f_i}{\partial l_i^2} < 0, \; f_{i\alpha_i} \equiv \frac{\partial f_i}{\partial \alpha_i} \geq 0, \; f_{i\alpha_i}' \equiv \frac{\partial^2 f_i}{\partial \alpha_i \partial l_i} \geq 0$$
$$\text{with strict inequality for at least one } i. \tag{13}$$

$$w' > 0 \qquad w' + l_i w'' > 0, \; i = 1, 2, \cdots, n. \tag{14}$$

If α_i is the amount of firm i_s' fixed capital, its increase may increase its total output as well as its marginal product of labor, satisfying (13) with strict inequality. Or if α_i is a parameter representing its technology level, the same inequality holds. Inequalities in (14) hold if the wage function is strictly increasing, convex function of L.

If the maximum is interior, and if, in addition, firm i behaves as a Cournot oligopsonist, its first order condition for profit maximization yields

$$\frac{\partial \pi_i}{\partial l_i} = pf_i'(l_i, \alpha_i) - \left(w(L) + l_i w'(L)\right) = 0, \quad i = 1, 2, \cdots, n. \tag{15}$$

Under (13) and (14), the second order condition is satisfied. Solving (15) with respect to x_i as a function of L and α_i,

$$l_i \equiv \psi^i(L, \alpha_i), \quad i = 1, 2, \cdots, n, \tag{16}$$

where

$$\frac{\partial \psi^i}{\partial L} \equiv \psi_L^i = \frac{w' + l_i w''}{pf_i'' - w'} < 0, \qquad i = 1, 2, \cdots, n. \tag{17.1}$$

$$\frac{\partial \psi^i}{\partial \alpha_i} \equiv \psi_{\alpha_i}^i = -\frac{f'_{i\alpha_i}}{pf_i'' - w'} \geq 0$$

$$\text{with strict inequality for at least one } i, \quad i = 1, 2, \cdots, n. \tag{17.2}$$

Let

$$\psi(L, \alpha_1, \cdots, \alpha_n) \equiv \sum_j \psi^j(L, \alpha_j).$$

Given $\alpha_i s$, the total demand for labor in equilibrium must be equal to the total supply of labor. Hence, in equilibrium,

$$w = w(L) \tag{18.1}$$

and

$$w = w(\psi(L, \alpha_1, \cdots, \alpha_n)) \equiv h(L, \alpha_1, \cdots, \alpha_n) \tag{18.2}$$

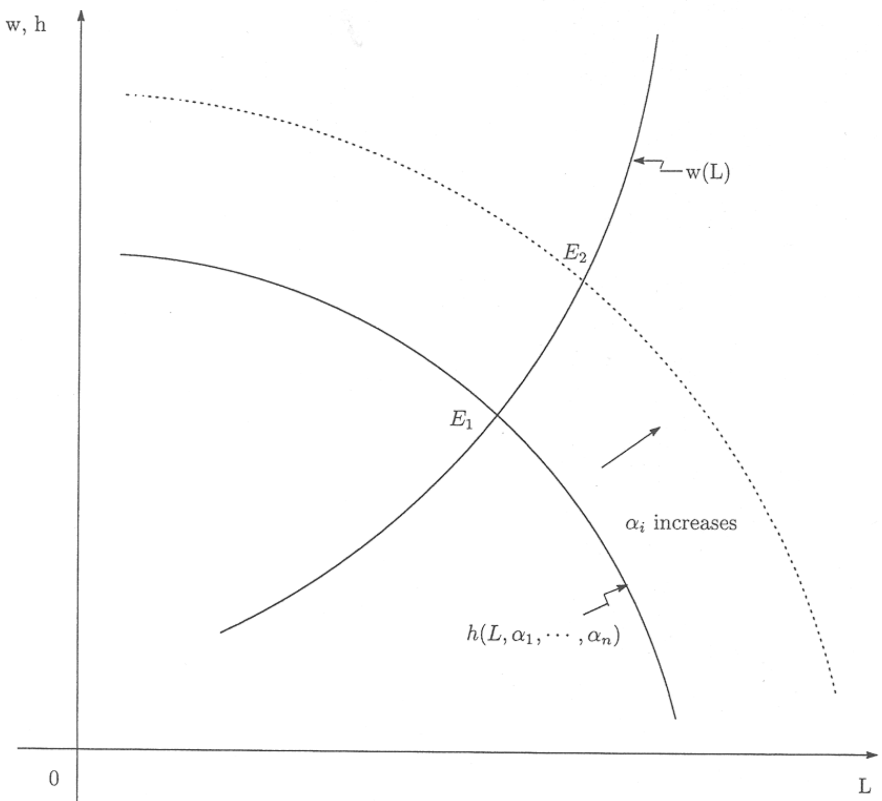
have to hold simultaneously, where

$$\frac{\partial h}{\partial L} = w' \frac{\partial \psi}{\partial L} < 0, \tag{19.1}$$

$$\frac{\partial h}{\partial \alpha_i} = w' \frac{\partial \psi_i}{\partial \alpha_i} \geq 0 \text{ with strict inequality for at least one } i. \tag{19.2}$$

Given $\alpha_i^{'}s$, the unique equilibrium for w and L is identifiable as the intersection E_1 of the downward-sloping curve for h and the upward-sloping one for $w(L)$ as in Figure 2. If α_i for which $f_{i\alpha_i}^{'} > 0$ holds increases, the curve for h shifts upward, resulting in a larger L and a higher w, which are shown by the new intersection E_2.

Figure 2: Equilibrium Wage Rate and Labor Supply.



Suppose next entry of a new firm. In this case the curve for

$$w\left(\psi\left(L, \alpha_1, \cdots, \alpha_n\right) + \psi^{n+1}\left(L, \alpha_{n+1}\right)\right)$$

lies above that for $w(\psi(L, \alpha_1, \cdots, \alpha_n))$. Hence, the new equilibrium values for w and L increase as in the case of an increase in α_i.

4 Cournot Oligopsonistic Oligopoly

In this section we consider oligopolistic firms producing homogeneous goods which are assumed to be the only demanders for the same factor of production. Firm $i's$ profit function is

$$\pi_i \equiv f_i(l_i, \alpha_i) p\left(\sum_j f_j\left(l_j, \alpha_j\right)\right) - l_i w\left(\sum_j l_j\right), \quad i = 1, 2, \cdots, n, \quad (20)$$

where the notation is defined as in Sections 2 and 3. Maximizing π_i with respect to l_i and assuming away a corner solution, we have

$$\begin{aligned} \frac{\partial \pi_i}{\partial l_i} &= f_i'\left(l_i, \alpha_i\right) p\left(X\right) + f_i'\left(l_i, \alpha_i\right) f_i\left(l_i, \alpha_i\right) p'\left(X\right) \\ &\quad - \left(w\left(L\right) + l_i w'\left(L\right)\right) = 0, \end{aligned} \quad (21)$$

where

$$X \equiv \sum_j f_j\left(l_j, \alpha_j\right), L \equiv \sum_j l_j. \quad (22)$$

We assume the second order condition to be satisfied.

$$\begin{aligned} \frac{\partial^2 \pi_i}{\partial l_i^2} &= f_i''\left(p + x_i p'\right) + \left(f_i'\right)^2\left(p' + x_i p''\right) + \left(f_i'\right)^2 p' \\ &\quad - \left(2w' + l_i w''\right) < 0, \quad i = 1, 2, \cdots, n. \end{aligned} \quad (23)$$

We note that (23) holds if

$$p + x_i p' \geq 0, \quad p' + x_i p'' < 0, \quad w'' \geq 0 \quad (23)'$$

that is, if any firm's marginal revenue is nonnegative and decreasing with respect to an increase in any other firm's output and if, in addition, the wage function is convex with respect to the total labor supply. We assume $(23)'$ to be true in the following analysis.

Solving (21) with respect to l_i, we get

$$l_i \equiv \psi^i\left(L, X, \alpha_i\right), i = 1, 2, \cdots, n. \quad (24)$$

where[2]

$$\frac{\partial \psi^i}{\partial L} < 0, \ \frac{\partial \psi^i}{\partial X} < 0, \ \frac{\partial \psi^i}{\partial \alpha_i} \geq 0, \ \ i = 1, 2, \cdots, n,$$
$$\text{with strict inequality for at least one } i. \tag{25}$$

The definition in (22) yields (26) and (27) below.

$$L = \sum_j \psi^j (L, X, \alpha_j) \equiv M_1 (L, X, \alpha_1, \cdots, \alpha_n), \tag{26}$$

where

$$\frac{\partial M_1}{\partial L} < 0, \ \frac{\partial M_1}{\partial X} < 0, \ \frac{\partial M_1}{\partial \alpha_i} \geq 0$$

[2]We can derive (25) as follows. Let the expression between $\frac{\partial \pi_i}{\partial l_i}$ and 0 in (21) be $g_i(l_i, L, X, \alpha_i)$. Then

$$g_i(l_i, L, X, \alpha_i) = 0.$$

Totally differentiating g_i,

$$g_{il_i} dl_i + g_{iL} dL + g_{iX} dX + g_{id_i} d\alpha_i = 0,$$

where

$$\begin{aligned}
g_{il_i} \equiv \frac{\partial g_i}{\partial l_i} &= f^{''}(p + x_i p^{'}) + (f_i^{'})^2 p^{'} - w^{'} < 0, \\
g_{iL} \equiv \frac{\partial g_i}{\partial L} &= -(w^{'} + l_i w^{''}) < 0, \\
g_{iX} \equiv \frac{\partial g_i}{\partial X} &= f_i^{'}(p^{'} + x_i p^{''}) < 0, \\
g_{i\alpha_i} \equiv \frac{\partial g_i}{\partial \alpha_i} &= f_{i\alpha_i}^{'}(p + x_i p^{'}) + f_i^{'} f_{i\alpha_i} p^{'} \geq 0, \\
&\quad \text{with strict inequality for at least one } i.
\end{aligned}$$

Hence

$$\begin{aligned}
\frac{\partial \psi^i}{\partial L} &= -\frac{g_{iL}}{g_{il_i}} < 0 \\
\frac{\partial \psi^i}{\partial X} &= -\frac{g_{iX}}{g_{il_i}} < 0 \\
\frac{\partial \psi^i}{\partial \alpha_i} &= -\frac{g_{i\alpha_i}}{g_{il_i}} \geq 0 \\
&\quad \text{with strict inequality for at least one } i.
\end{aligned}$$

This proves (25).

with strict inequality for at least one i. $(26)'$

$$X = \sum_j f_j \left(\psi^j (L, X, \alpha_j), \alpha_j\right) \equiv M_2 (L, X, \alpha_1, \cdots \alpha_n), \tag{27}$$

where

$$\frac{\partial M_2}{\partial L} < 0, \ \frac{\partial M_2}{\partial X} < 0, \ \frac{\partial M_2}{\partial \alpha_i} \geq 0$$

with strict inequality for at least one i. $(27)'$

The Cournot equilibrium values of X and L are defined by (26) and (27). Solving (26) with respect to L, we have[3]

$$L \equiv N_1(X, \alpha_1, \cdots, \alpha_n), \tag{28}$$

where

$$\frac{\partial N_1}{\partial X} < 0, \ \frac{\partial N_1}{\partial \alpha_i} \geq 0 \text{ with strict inequality for at least one } i. \qquad (28)'$$

On the other hand, (27) leads to

$$X \equiv N_2(L, \alpha_1, \cdots \alpha_n), \tag{29}$$

where

$$\frac{\partial N_2}{\partial L} < 0, \ \frac{\partial N_2}{\partial \alpha_i} \geq 0 \text{ with strict inequality for at least one } i. \qquad (29)'$$

[3]Totally differentiating (26), rearranging and letting $d\alpha_i \neq= 0, d\alpha_j = 0$, for all $j \neq i$, we have

$$\frac{\partial N_1}{\partial X} = \frac{M_{1L}}{1 - M_{1L}} < 0$$

$$\frac{\partial N_1}{\partial \alpha_i} = \frac{M_{1\alpha_i}}{1 - M_{1L}} \geq 0$$

with strict inequality for at least one i.

This establishes $(28)'$.

Before proceeding further, assume[4] that for all $L > 0$,

$$\left.\frac{dX}{dL}\right| L = M_1(\cdot) = \frac{1 - M_{1L}}{M_{1X}} < \frac{M_{2L}}{1 - M_{2X}} = \left.\frac{dX}{dL}\right| X = M_2(\cdot), \tag{30}$$

that is,

$$A \equiv \begin{vmatrix} 1 - M_{1L} & M_{1X} \\ M_{2L} & 1 - M_{2X} \end{vmatrix} > 0. \tag{30$'$}$$

Under $(A.30')$, given $\alpha_i^{'}s$, the equilibrium is given by the intersection E_1 of the two downward-sloping curves as in Figure 3.

Suppose next that α_i (or n, the number of firms) increase. Then the two curves shift upward. The new equilibrium entails a larger equilibrium value for at least one variable (L or X) but the equilibrium value for the other variable may be equal to, larger or lower than before the change in α_i or n[5 6]. In Figure 3. the new equilibrium E_2 leads to larger equilibrium values for both L and X.

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[4]Let

$$\begin{aligned} G_1(L, X; \bullet) &\equiv L - N_1(X; \bullet) = 0 \\ G_2(L, X; \bullet) &\equiv X - N_2(L; \bullet) = 0 \end{aligned}$$

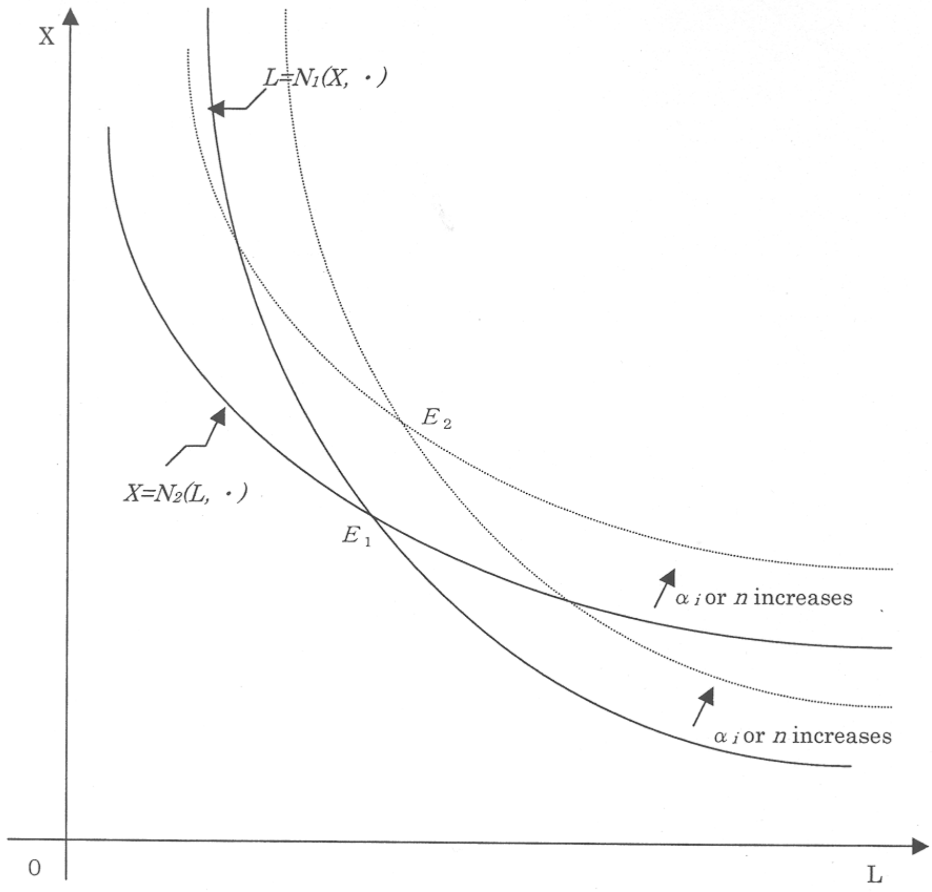
Under $(30)'$, the Jacobian matrix of (G_1, G_2) is ensured to be a P-matrix in the sense of Gale and Nikaido (1965). Hence the above system of equations has a globally univalent solution. In order to further clarify the economic implication of $(30)'$, let the product market be perfectly competitive as in Section 3. As $M_{1X} = 0, M_{2X} = 0$ in this case, $(30)'$ becomes equivalent to $1 - M_{1L} > 0$, which certainly holds if $w' > 0, w'' \geq 0$ and $f_i'' < 0$.

[5]The ambiguity regarding the effects of a change in α_i can be analytically shown by the following expressions derived from (28) and (29).

$$\begin{aligned} \frac{\partial L}{\partial \alpha_i} &= \frac{M_{1\alpha_i}(1 - M_{2X}) + M_{1X} M_{2\alpha_i}}{A}, \\ \frac{\partial X}{\partial \alpha_i} &= \frac{M_{2\alpha_i}(1 - M_{1X}) + M_{2L} M_{1\alpha_i}}{A}. \end{aligned}$$

[6]Intuitively, entry may lead to a higher equilibrium factor price. Our analysis here, however, suggests that this is not necessarily the case. Suppose entry occur. Its in mediate effects would be a higher factor price, a larger industry output and a lower

Figure 3: Equilibrium Industry Output and Labor Supply.



5 Conclusion

In this paper we have proven under reasonable, general assumptions the existence of a unique equilibrium for three Cournot models for imperfect competition, that is, Cournot oligopoly, Cournot oligopsony and Cournot oligopsonistic oligopoly. We have been able to show diagrammatically existence of the equilibrium aggregate quantity and price simultaneously for the first two models, while we have presented a diagrammatic method for proving simultaneously existence of the equilibrium industry output and factor demand for the third model[7]. We have derived definite comparative static results for Cournot oligopoly and Cournot oligopsony. In the case of Cournot oligopsonistic oligopoly, however, the effects on the equilibrium aggregate variables of a change in a parameter or the effects of entry have been found to be ambiguous.

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product price. This in turn cause firms with lower technology level to decrease factor demands. On the other hand, firms with advantageous technology may increase or decrease their factor demands. In the second case, the total industry factor demand decreases in the event of entry. In the first case it may increase or decrease. If the increased factor demands by the advantageous firms are small, the total industry factor demand may decrease.

[7]Okuguchi (1998) has proven existence of the Cournot oligopsonistic oligopoly equilibrium for a more general model with two factors of production. He has reduced the existence problem to a fixed point problem for the industry output alone. However, he has not considered simultaneous determination of the equilibrium industry output and factor demands.

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